

# Centroidal Momentum Based Trajectory Generation and Implementation on Legged Robots

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**Abstract**—This research explores the potential of the *Centroidal Momentum* (CM) of a legged robot in developing an aperiodic trajectory generation algorithm that is computationally light. The resulting trajectories are validated on a robot testbed with two quasi-direct-drive legs [1].

## I. INTRODUCTION

While steady-state periodical motions based on *a priori* off-line computation have been implemented on many robots, the pursuit for an computationally light algorithm for aperiodic motions remains largely an open question. The difficulty is partly rooted in a robot’s large number of degrees-of-freedom (DoF). Methods based on full body dynamics [2] tend to suffer from extensive computation time. A simplified model which ignores leg inertia is used on MIT Cheetah 2 [3]. With the robot’s ability of proprioceptive ground reaction forces (GRF) control, such a model is capable of online planning for autonomous running and jumping.

Our lab have developed a miniature motor module with high torque density and low gear ratio shown in Figure 1. The quasi-direct-drive legs are of low impedance thus allow for direct control over GRF, similar to that of the MIT Cheetah 2. With its ability to execute prescribed force commands, the robot’s *centroidal momentum* (CM) can be directly obtained. We propose to incorporate the robot’s CM information in trajectory generation. This method should produce more precise result when planning motions for models with massive legs. A series of jumping motions have been performed as validations of the proposed method.



Fig. 1. Two quasi-direct-drive legs on our robot testbed

## II. PROPOSED APPROACH

Our approach is based on the following observations: 1) a low impedance leg empowers prescribed force profile control, 2) given GRF, the closed-form solutions of a robot’s CM can be computed, 3) the CM can also be calculated from  $h_G = A_G(q)\dot{q}$ , where  $A_G(q)$  is the *Centroidal Momentum Matrix* (CMM). We present an optimization scheme that treats GRF as optimization variables, and at the same time optimizes over all actuated joints with a constraint that the CM calculated from generalized velocities should agree with that from the

force profile. It is worth noting that such constraints are often not sufficient to recover all the states. This provides us with the flexibility to manipulate the rest of the DoFs for additional goals. Such a formulation is guaranteed to produce dynamically feasible motions.

The proposed method has been used to create some typical locomotions such as jumping and bounding on a 7-DoF two-legged planar model. The outputs of this method include all joint angles and torques. Such rich information is valuable in enforcing kinematic constraints and designing cost function. It also allows incorporation of actuator saturations. All aforementioned calculations can be done under practical limitations of computation time and power.

## III. CURRENT RESULTS AND POSSIBLE OUTCOME

We have been implementing some generated trajectories to a 7-DoF robot testbed. A typical leaping motion is shown in Figure 2. The body plate is attached to a carbon fiber tube through a bearing to restrict its motion in the sagittal plane without affecting its pitch angle.

Horizontal leaps up to 60 cm (three times the body length) and vertical jumps up to 40cm have been achieved. These results match to the prediction of our simulation. The proposed approach will be implemented on a quadrupedal robot for more complex 3D motions.

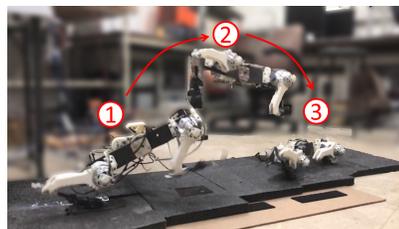


Fig. 2. A leaping created with the proposed trajectory optimization method. 1: stance, 2: aerial, 3: landing

## REFERENCES

- [1] Yanran Ding and Hae-Won Park. Design and experimental implementation of a quasi-direct-drive leg for optimized jumping. In *2017 IEEE/RSJ International Conference on Intelligent Robots and Systems*, Accepted in 2017.
- [2] Katja Mombaur. Using optimization to create self-stable human-like running. *Robotica*, 27(03):321–330, 2009.
- [3] Hae-Won Park, Patrick M Wensing, Sangbae Kim, et al. Online planning for autonomous running jumps over obstacles in high-speed quadrupeds. 2015.